

Spinorial space-time and privileged space direction (I)

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Abstract

Contrary to frequent statements, the indications from WMAP and Planck data analyses suggesting the possible existence of a privileged space direction are not strange phenomena from the point of view of fundamental space-time geometry. As already emphasized in our previous papers on the subject, the spinorial space-time we suggested in 1996 naturally incorporates a privileged space direction for each comoving observer. This space direction is simply defined by the sigma-like matrix of which the cosmic space-time position spinor is an eigenspinor. In this series of papers, we remind some of our previous work and discuss the present situation.

1 Introduction

The ultimate structure of space-time remains a fundamental issue in Particle Physics as well as in Cosmology. A possibility suggested in [1, 2] was to replace the standard real four-dimensional space-time by the complex two-dimensional one actually felt by spin-1/2 particles. Then, for space dimensions, the standard rotation group $SO(3)$ must be replaced by $SU(2)$ so that spin-1/2 particles can be representations of the actual group of space-time transformations.

Given the existence of comoving frames, the use of relativity at the overall cosmic level is not compelling at that stage. It may even happen that relativity just corresponds to the structure of space-time as seen locally by standard matter at low energy [3]. Indeed, the spinorial space-time does not require from the beginning the use of any critical speed.

Extracting from a cosmic (space-time position) spinor ξ the scalar $|\xi|^2 = \xi^\dagger \xi$ where the dagger stands for hermitic conjugate, a positive cosmic time $t = |\xi|$ can be defined which leads in particular to a naturally expanding Universe. Other definitions of cosmic time using the same $SU(2)$ scalar produce in practice similar results [4].

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In such an expanding Universe, the point $\xi = 0$ plays the role of a cosmic time origin (the birth of the Universe) and the $|\xi| = t_0$ hypersphere describes the space at a given time t_0 .

Then, for any space-time position spinor ξ the space translations are given by the SU(2) transformations of the spinor. The origin $\xi = 0$ remains invariant. Relative space coordinates are defined in this way on the constant-time S^3 hypersphere. Space rotations of these coordinates around a fixed space-time point ξ_0 taken to be the origin in the spatial hypersphere are SU(2) transformations acting on the SU(2) group itself.

A privileged space direction associated to ξ is then defined by the linear combination of sigma matrices (with real coefficients) that leaves ξ invariant. The space-time points on the trajectory generated by this sigma-like matrix satisfy the relation $\xi' = \exp(i\phi) \xi$ where ϕ is real and $\exp(i\phi)$ is a complex phase factor. Such a definition is stable under the relevant SU(2) transformations. It is also invariant under comoving time evolution.

An equivalent definition of this privileged direction is provided by $|\xi|^{-2} \xi^\dagger \vec{\sigma} \xi$ where $\vec{\sigma}$ is the vector formed by the three Pauli matrices. The value of the vector $|\xi|^{-2} \xi^\dagger \vec{\sigma} \xi$ is common to all points lying on the privileged space trajectory.

Together with parity violation, this property of the spinorial space-time can be at the origin of the asymmetry in the average temperatures on opposite hemispheres of the sky (defining therefore a privileged space direction) as possibly confirmed by Planck [5] after previous WMAP observations.

The possible existence of related phenomena in the ultra-high energy cosmic-ray (UHECR) flux and in other UHECR properties is also worth exploring. Similarly, for all potential cosmic-ray and accelerator signatures of the spinorial space-time.

Here follows the 22 September 2011 *Post Scriptum* to our paper *Cosmic rays and tests of fundamental principles*, arXiv:1011.4889 version v4, <http://arxiv.org/abs/1011.4889v4> (pages 7 - 11), discussing in detail the existence of a privileged space direction in the spinorial space-time as well as other related items. Unappropriate sub-subsection numbers 6.2.a to 6.2.c have been replaced by 6.1.a to 6.1.c in order to consistently organize the subsections. The reference list is adapted to the present article.

2 Post Scriptum of 22 September 2011 to paper arXiv:1011.4889 (CRIS 2010 Conference) after publication of the initial article in Nuclear Physics Proceedings

(Section 6 of the original paper)

6.1 SL(2,C), SU(2), SO(4), causality, half-integer spin and Cosmology

To define local space coordinates in the above spinorial approach to space-time [2], one can

consider a spinor ξ_0 (the observer position) on the $|\xi| = t_0$ hypersphere. Writing, for a point ξ of the same spatial hypersphere :

$$\xi = U \xi_0 \quad (1)$$

where U is a cosmic $SU(2)$ transformation :

$$U = \exp(i/2 t_0^{-1} \vec{\sigma} \cdot \vec{x}) \equiv U(\vec{x}) \quad (2)$$

and $\vec{\sigma}$ the vector formed by the Pauli matrices, the vector $\vec{x}$, with $0 \leq x$ (modulus of $\vec{x}$) $\leq 2\pi t_0$, can be interpreted as the cosmic spatial position vector of ξ with respect to ξ_0 at constant time t_0 .

A 2π $SU(2)$ rotation in the spinorial space-time ($x = 2\pi t_0$, $U = -1$), or the equivalent change of coordinates, changes the signs of ξ_0 and ξ simultaneously. It therefore turns ξ_0 and ξ into their cosmic antipodals but leaves $\vec{x}$ invariant. Thus, the relevant variation domains for x before reaching again the identity map are $0 \leq x \leq 2\pi t_0$ in the case of space coordinates defined at constant t_0 , and $0 \leq x \leq 4\pi t_0$ for space-time spinors.

Conventional ("euclidean") local space coordinates are obtained for $x \ll t_0$ where the effects of the noncommutativity of the Pauli matrices can be neglected.

Thus, space translations are just $SU(2)$ transformations acting on the space-time spinors whereas standard space rotations act on the $SU(2)$ transformations themselves.

6.1.a A simple cosmological check

Assuming that the $|\xi| = t_0$ hypersphere corresponds to the three-dimensional space when the age of our Universe is t_0 naturally leads to the so-called Hubble's law, actually first formulated by Georges Lemaître [6], relating distances and relative velocities.

More precisely, assuming that the ratio between the physical space units and time units remains constant with cosmic time, and that the cosmic time units correspond to the time units of observations, the value of the Hubble (Lemaître) constant would be exactly equal to the inverse of the age of the Universe t_0 . The expansion speed would then be constant with time (vanishing deceleration parameter). Similarly, redshifts are automatically generated by the expansion of the Universe in the spinorial space-time.

Such a structure is obtained on very basic and general geometric grounds, without introducing any specific cosmology and with even no space units or Minkowskian metric. At that stage, the four components of the two complex spinorial coordinates are measured with a single (time) scale, and the only specific space coordinates are $SU(2)$ -angular and comoving.

Actually, the large-scale structure of our Universe appears compatible with a $SO(4)$ symmetry where, instead of being imaginary, the cosmic time would correspond to the modulus of a four-vector. $SO(4)$ would unify, in this context, translations in the curved space with the standard space rotations. However, as shown above and further discussed below, translations and space rotations should actually be viewed as different representations of a single $SU(2)$ group.

A transition from such a cosmic $SU(2)$ or $SO(4)$ inherited from the initial singularity $\xi = 0$ to a "dynamical" symmetry including $SL(2, C)$ at the nowadays local space-time scales is obviously possible, but it would require further physical and geometrical input.

Thus, we have produced in a very simple way the pattern of a potentially ever-expanding universe with positive (3-spherical) curvature that may be basically compatible with experimental data and deserves being further explored. Contrary to cosmologies where a fourth Euclidean coordinate is related to an imaginary time, a cosmic time is defined here as the modulus of a $SU(2)$ spinor or that of the associated real four-vector.

As we have not yet introduced in the model matter and its interactions, or even the precise connection between space units and angular commoving coordinates at a given time, we shall not try to discuss here topics like the possible acceleration of the expansion of the Universe, dark matter or dark energy. We defer this to a later work.

Light waves for a zero-mass photon with a critical speed c can be introduced without any explicit reference to Lorentz symmetry, just by defining the energy as the product of the speed of light by the inverse of the wavelength. The need for standard relativity arises when considering massive conventional particles of light emission by moving sources.

6.1.b Other space-time transformations

Using the above conventions, a standard space rotation around ξ_0 is defined by a $SU(2)$ element $U(\vec{y})$ turning any $U(\vec{x})$ into $U(\vec{y}) U(\vec{x}) U(\vec{y})^\dagger$. The vector $\vec{y}$ provides the rotation axis and angle. As already stated, a 2π rotation ($x = 2\pi t_0$) leaves $\vec{x}$ invariant.

Translations are therefore "cosmic" rotations of the spinors around the "initial" point of the Universe $\xi = 0$, whereas standard space rotations are $SU(2)$ transformations acting on the set of local space translations from a fixed point ξ_0 .

But another kind of local position can also be defined if the requirement of constant cosmic time does not apply to the path between ξ_0 and ξ : the "direct" spinorial separation $\Delta\xi = \xi - \xi_0$, which changes sign under a 2π $SU(2)$ rotation in the spinorial space-time. A spinorial $SU(2)$ rotation around ξ_0 is then constructed by transforming the point ξ into $\xi' = \xi_0 + U(\vec{y}) \Delta\xi$. Obviously, the spinorial position behaves differently from the standard space position under a 2π rotation. As the spinorial position can in principle be used to build wave functions similar to the standard vector position, we should expect half-integer angular momenta to be realized in Nature.

It must however be noticed that the fact that such a spinorial position is not defined through a constant-time link may have important negative consequences potentially limiting its role in standard physics, even if the position $\Delta\xi$ clearly exists from a mathematical point of view and transforms like a spinor under $SU(2)$ rotations. Strictly speaking, a "straight line" between ξ_0 and ξ in the spinorial space-time would not be really compatible with standard causality in our classical macroscopic cosmic picture, due to the very small effect from the Universe spatial curvature. Contrary to the standard space position, a $SU(2)$ spinorial rotation around ξ_0 does not necessarily transform ξ into an element of the $|\xi'| = t_0$ hypersphere. The image of

$\Delta\xi$ under $SU(2)$ transformations is actually a S^3 hypersphere similar to the spinorial Universe itself but centered around ξ_0 .

More generally, any effective space curvature at constant time, even local, can potentially lead to the same kind of phenomenon and obstruct the "direct" spinorial link.

This circumstance may explain the absence in Nature of half-integer angular momenta other than those associated to the spin of "elementary" fermions. One can conceive that the "direct" spinorial position is relevant only at extremely small distance scales, where the internal structure of conventional "elementary" particles is formed (Planck or beyond ?) and standard causality or similar constraints, including the arrow of time, do no longer hold. The constant-time space coordinates considered above cannot lead to a half-integer spin.

Thus, the spin $1/2$ could in principle be naturally generated as angular momentum of the matter in motion forming our standard particles at distance scales where causality and other standard requirements do not apply and the description of kinematics can be different ("inside" an electron or a quark).

The existence of half-integer spin may then be interpreted as a possible evidence for causality-violating physics (or some equivalent change in the effective space-time structure felt by matter) at ultimate length and space scales, if one assumes that such a spin is indeed some sort of physical "orbital" angular momentum corresponding to the fundamental representation of $SU(2)$. Quantum uncertainties or pre-quantum physics and similar phenomena can naturally play a role at these very small scales, even if half-integer spin is not generated in the conventional quantum mechanics using standard space coordinates and rotations based on our macroscopic perception of space and time.

Similar considerations also suggest that the region between $\xi = 0$ and, roughly, the $\xi = t_{Planck}$ (Planck time) hypersphere can naturally correspond to a pre-Big Bang scenario taking $\xi = 0$ as the initial singularity or starting from a (superbradyonic ?) time scale much smaller than the Planck time. Contrary to existing prejudices, tests of pre-Big Bang physics can in principle be possible with WMAP and Planck data, but also through UHECR experiments where extrapolations from physics beyond Planck scale may be detectable.

In our $SU(2)$ Universe, $SL(2,C)$ transformations introduce basically a change of relative scale between the two components of the cosmic spinor taking the $SL(2,C)$ transformation to be diagonal. Similar transformations can be considered if the origin is the point ξ_0 instead of $\xi = 0$.

6.1.c A privileged space direction ?

A specific property of the spinorial space-time considered here is that, to each point ξ , a (complex) one-dimensional spinorial subspace can be associated such that for any point ξ' of this subspace one has :

$$\xi^\dagger \xi' = |\xi'| \cdot |\xi| \exp(i\phi) \quad (3)$$

where $\exp(i\phi)$, with ϕ real, stands for a complex phase.

If $|\xi'| = |\xi|$ so that ξ and ξ' belong to the same constant-time hypersphere, the complex phase $\exp(i\phi)$ is associated to the SU(2) matrix transforming ξ into ξ' . This transformation, $U = \exp(i/2 t^{-1} \vec{\sigma} \cdot \vec{z})$ where t is the cosmic time $t = |\xi|$ and $\vec{z}$ a real space vector, is generated by a sigma-like matrix $\vec{\sigma} \cdot \vec{z} / |z|^{-1}$ associated to a unique space direction $\vec{z} / |z|^{-1}$ on the constant time hypersphere.

ξ and ξ' are both eigenspinors of $\vec{\sigma} \cdot \vec{z}$. For each point ξ of the spinorial space-time, other than $\xi = 0$, there exists a unique space direction for which ξ is an eigenspinor of the associated sigma-like matrix. Exponentiating this matrix with an imaginary coefficient generates the directions of the relevant (complex one-dimensional) spinorial subspace associated to ξ .

With $|\xi'| = |\xi|$ and a positive phase ϕ , one actually has :

$$\xi' = \exp(i/2 t^{-1} |z|) \xi \quad (4)$$

and similarly, with $-|z|$ instead of $|z|$, if ϕ is negative.

The set of points of the spinorial space-time thus generated obviously corresponds to a (spinorial) circle of radius $|\xi'| = t$ ($t =$ cosmic time) on the constant-time hypersphere, including the point ξ itself and its SU(2) antipodal - ξ .

Thus, "looking at" the initial point of our Universe $\xi = 0$ from a point ξ of the present time spatial hypersphere naturally leads, in the spinorial coordinates considered here, to the definition of a privileged space direction on the space hypersphere itself.

The direct memory of the geometry leading to such a privileged space direction is basically lost if standard space coordinates on the constant-time hypersphere are used and standard matter is dealt with without incorporating its deepest structure as well as the most primordial origin of the Universe. However, several possible tracks from this spinorial effect in Cosmology and Particle Physics can still be considered.

In particular :

- The internal structure of standard spin-1/2 particles, as well as their interaction properties at very small distance scales, may contain the expression of a similar phenomenon.
- Signatures from a pre-Big Bang era can yield relevant information on this privileged space direction and on effects of the same origin through WMAP, Planck and other experiments.
- Similarly, ultra-high energy cosmic rays may be sensitive to both cosmological and "beyond Planck" phenomena containing effects related to the privileged space direction.

Further work on this subject is clearly required.

6.2 From SL(2,C) to SL(N,C) ?

To the above SU(2) symmetry, a natural SL(2,C) is tacitly associated, potentially including Lorentz transformations in the usual way except possibly for the definition of the time variable.

This point will be further discussed elsewhere. Then, $|\xi|^2 = \xi^\dagger \xi$ is no longer a scalar, and cosmic time can be defined only in a preferred reference frame. Lorentz symmetry may be preserved in the limit where Physics does not depend on the choice of this frame and on the above definition of the cosmic time.

As in this approach space translations and rotations are not fundamentally different transformations, the question whether some suitable extension of $SL(2, \mathbb{C})$ can meet less stringent no-go constraints than those stated some decades ago [7] deserves being addressed.

We suggest, in particular, to consider in this kind of picture some broken version of $SL(N, \mathbb{C})$ as the possible global symmetry of the Physics of standard particles.

In a simple picture, $SL(N, \mathbb{C})$ would act on the two complex dimensions associated to the usual space-time, plus $N-2$ complex spinorial coordinates related to extra dimensions and to internal symmetries. N -spinors with nonvanishing coordinates only in the $N-2$ "non space-time" dimensions would be scalars under the standard space-time $SL(2, \mathbb{C})$. Transformations inside the new $N-2$ dimensions would correspond to internal symmetries.

Similarly, transformations between the standard $SL(2, \mathbb{C})$ space-time spinors and the spinors oriented in the new $N-2$ dimensions would be of supersymmetric nature.

A more detailed discussion of $SL(N, \mathbb{C})$ potentialities will be presented in a forthcoming paper.

6.3 $SO(N)$ and other groups

Similar to the $SO(4)$ discussed above, other $SO(N)$ symmetries can be considered as extensions of the pattern presented.

The geometric universe considered using $SU(2)$ and possibly $SO(4)$ can also be identified to the subset of null (zero-norm) vectors in a $SO(4,1)$ pattern where the fifth dimension would be the cosmic time, and the metric :

$$X^2 = \xi^\dagger \xi - t^2 \quad (5)$$

where X is the 5-vector formed by the four real components of the space-time spinor, and t the cosmic time defined as the modulus of this space-time spinor.

Local versions of the same symmetry centered around a $SU(2)$ spinorial point ξ_0 different from $\xi = 0$ also deserve consideration, even if in this case the direct physical interpretation the $|\xi - \xi_0|$ can appear much less obvious. The question of whether $|\xi - \xi_0|$ can be interpreted at very short distances as the "internal time" of an "elementary" particle deserves being addressed.

In the $SO(4,1)$ associated to the metric defined by (11), invariances of the Lorentz type naturally appear writing for instance :

$$x'_1 = \gamma x_1 - (\gamma^2 - 1)^{1/2} t \quad (6)$$

$$t' = \gamma t - (\gamma^2 - 1)^{1/2} x_1 \quad (7)$$

where x_1 is one of the four real coordinates extracted from the spinor ξ and x'_1, t' the transformed values of x_1, t . γ is taken to be real and positive, with $\gamma > 1$. No reference to a critical speed, nor even to space units, is required in the formulation of such trivial Lorentz-like transformations.

6.4 What does the concept of symmetry become at ultra-high energy ?

Is the Planck scale the ultimate limit where all symmetries are unified and become exact ? This standard picture is often considered as natural, and even the deformations of relativity by quantum gravity are turned into a new (deformed) symmetry where all inertial frames are equivalent. The possibility that new Physics can exist beyond the Planck scale is often just neglected.

However, in our Universe there is a natural reference frame provided by the cosmic microwave background radiation. Furthermore, it is by now impossible to exclude on experimental grounds a real breaking of Lorentz symmetry as considered in this paper and in the references quoted above. There is also by now no proof of the validity of currently fashionable grand-unification patterns, and no supersymmetric particle has yet been found in spite of many expectations.

Furthermore, we have just suggested a pre-cosmological geometry that reasonably incorporates several main features of the large-scale properties of our Universe without introducing standard matter and its interactions. It may therefore happen that standard particle physics is not at the origin of the deepest structure of the Universe at very large scales, and that it simultaneously fails to account for the properties of matter at the really fundamental level.

An alternative view to the standard picture of particle symmetries has been suggested in the Post Scripta to [8] and [9]. If the standard particles have a composite origin and the vacuum is not made of conventional matter, it may happen that the usual gauge principle has no real physical meaning at the ultimate fundamental scale and that the gauge bosons are generated by the fundamental matter only in specific situations involving the physical presence of conventional particles as vacuum excitations. Similarly, the Higgs boson would not need to be statically materialized in vacuum as a permanent condensate in the absence of surrounding standard particles.

In such a scenario, as energy increases beyond some scale that starts being sensitive to the internal structure of the standard "elementary" particles, differences between kinds of particles can become more and more detectable. The apparent symmetry may then tend to disappear or remain only as a low-energy approximation. Similarly, as considered above, the vacuum may start developing unexpected properties and other fundamental principles can possibly fail.

In other words, the standard concept of symmetry would be a mathematical construction to account for the intrinsic limitations of our low-energy view of matter. Similarly, for quantum field theory. Particles whose internal differences at a smaller distance scale cannot be observed by low-energy physics are described as "identical" in the sense of symmetry (including the generation pattern), even if they are actually not when examined at a deeper level.

Then, instead of considering *a priori* that symmetries will become more and more exact as the energy scale increases and masses can be neglected as compared to the energy scale, one can conceive that this behaviour of the laws of Physics will exist in some range of scales but will change above a transition energy scale. Above the transition energy, observations would then become sensitive to new features of the particle internal structure and to properties of the real fundamental matter, increasingly unveiling differences in structure among particles from the same symmetry multiplet. Just below this transition scale, standard symmetries will reach their maximal reliability and precision.

Such an energy dependence of the properties of particle symmetries would also help to escape strong no-go conditions on the content of the allowed symmetries of conventional particles. From this point of view, a $SL(N,C)$ symmetry can be considered as a tool and as a framework where to examine representations, symmetry breaking patterns, dynamical concepts and other relevant properties of Physics on a large set of scales, including practical no-go constraints.

(end of the *Post Scriptum* to paper arXiv:1011.4889)

3 Complementary remarks

The issues discussed in this *Post Scriptum* to paper arXiv:1011.4889 will be dealt with in more detail in a forthcoming article. But it seems relevant to emphasize here that the possibility to escape standard no-go theorems on symmetries may have implications for the highest-energy accelerator experiments.

It should also be noticed that the obvious relation $|\xi|^2 = |\xi^\dagger \vec{\sigma} \xi|$ is similar to a Lorentz-like metric. This parallelism deserves closer attention in connection with the space-time properties of matter. More generally, building particle wave functions with a spinorial space-time appears straightforward once the basic dynamics is defined.

Remarkably enough, a direct link beyond standard models may exist between the space-time structure of the Universe at the largest distance and time scales and that of matter at the smallest scales. The spinorial space-time can make this link explicit.

The spinorial space-time is likely to be the expression of a new fundamental vacuum structure where standard quantum field theory does not necessarily apply. The cosmological constant problem can possibly be solved in this way [10].

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